Simultaneous estimation of spatial and kinetic parameters in single rotation dynamic myocardial SPECT

Jonathan S. Maltz[†‡], Elijah Polak[‡] and Thomas F. Budinger[†‡]

[†]Center for Functional Imaging, LBNL

[‡]Department of Electrical Engineering and Computer Sciences, University of California at Berkeley

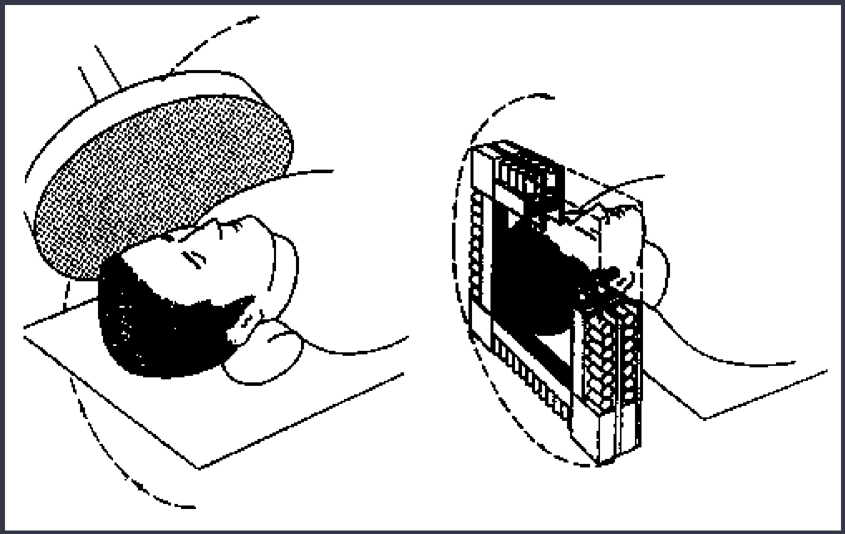
Outline

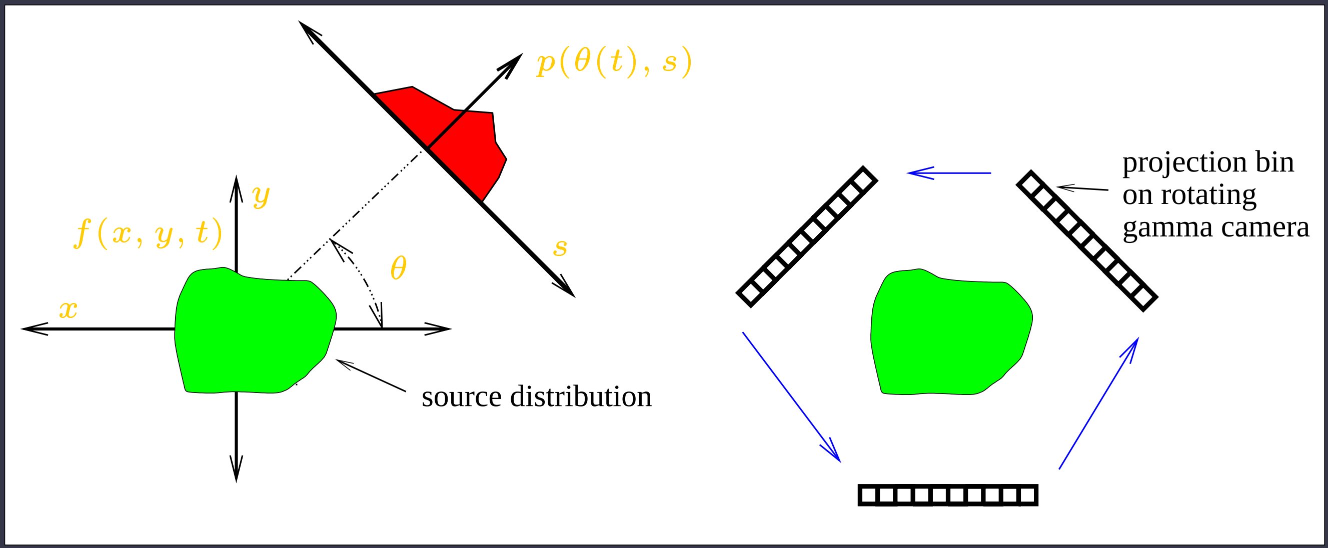
- Problem statement
- Previous work in simultaneous spatio-kinetic estimation in dynamic ECT
- Our approach
- Results
- Conclusions

Dynamic SPECT problem: Reconstruction directly from projections

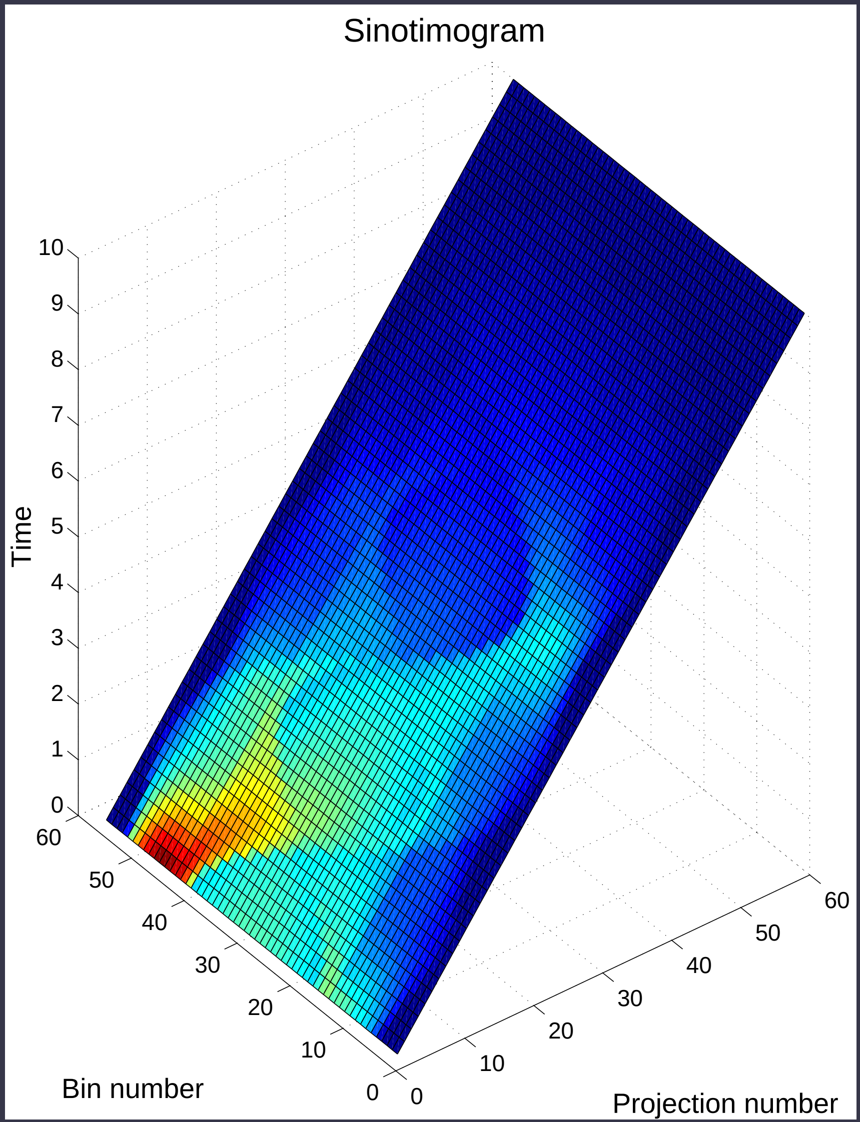
- Source distribution changes as camera rotates
- Incomplete spatio-temporal sampling of Radon transform
- Reconstruction requires estimation of both:
  - Geometry of the source distribution
  - Parameters of tracer kinetic model
- Very difficult, ill-conditioned, non-linear problem
- Cannot be assured of a unique solution owing to fundamental nature of problem

Imaging geometry





Interpolation of sinotimogram



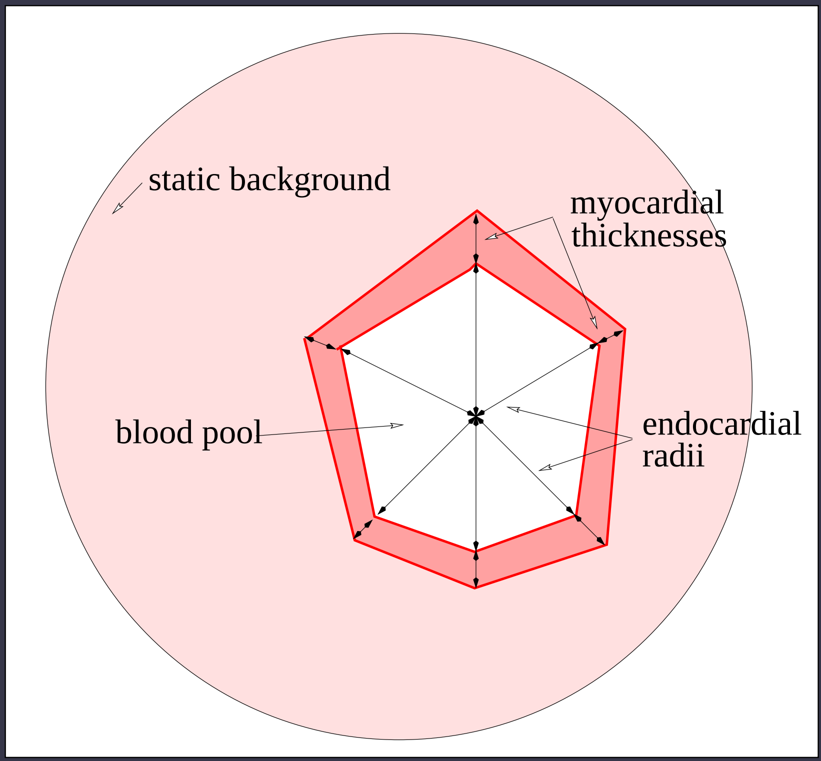
Previous work

- Many algorithms have been developed over the past two decades
- None has satisfactorily addressed the problem when:
  - The projection data contain realistic levels of noise

    AND
  - Several regions of differing kinetics are present among the sources

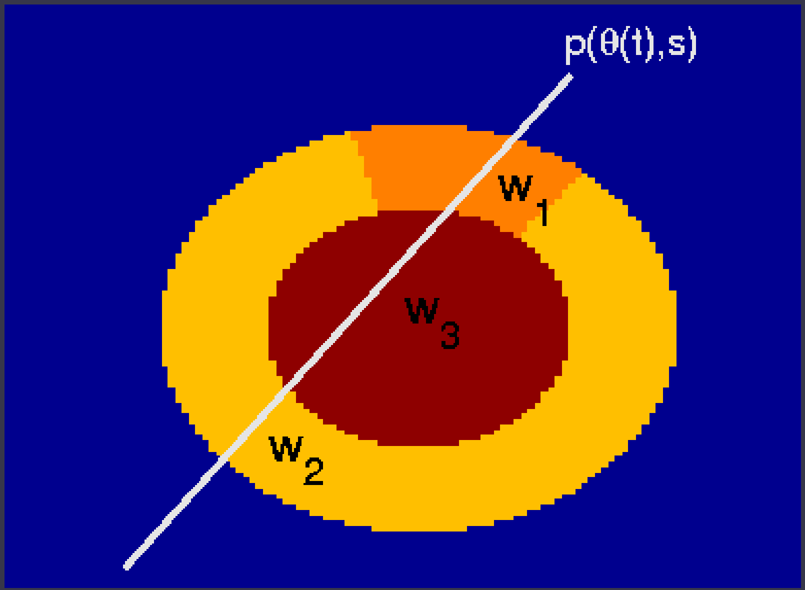
Simultaneous spatial and kinetic parameter estimation:

- We extend the joint spatial and kinetic parameter estimation approach of Chiao et al. (1994) to multiple dynamic regions

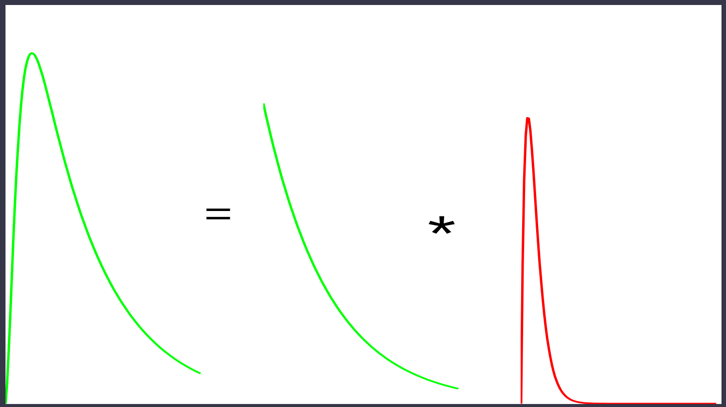


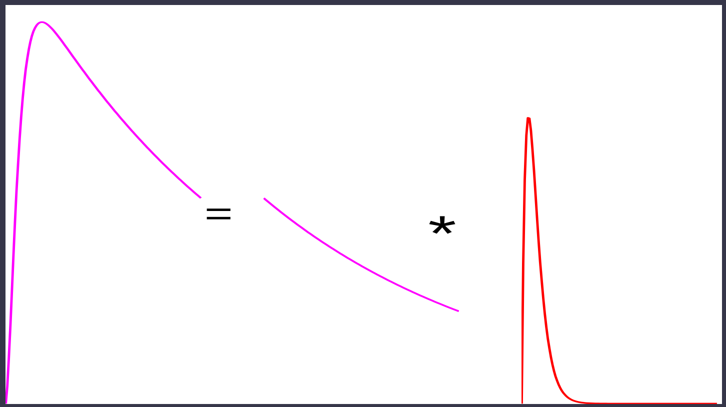
Chiao, Rogers, Clinthorne, Fessler and Hero, IEEE TMI 13(2) 1994

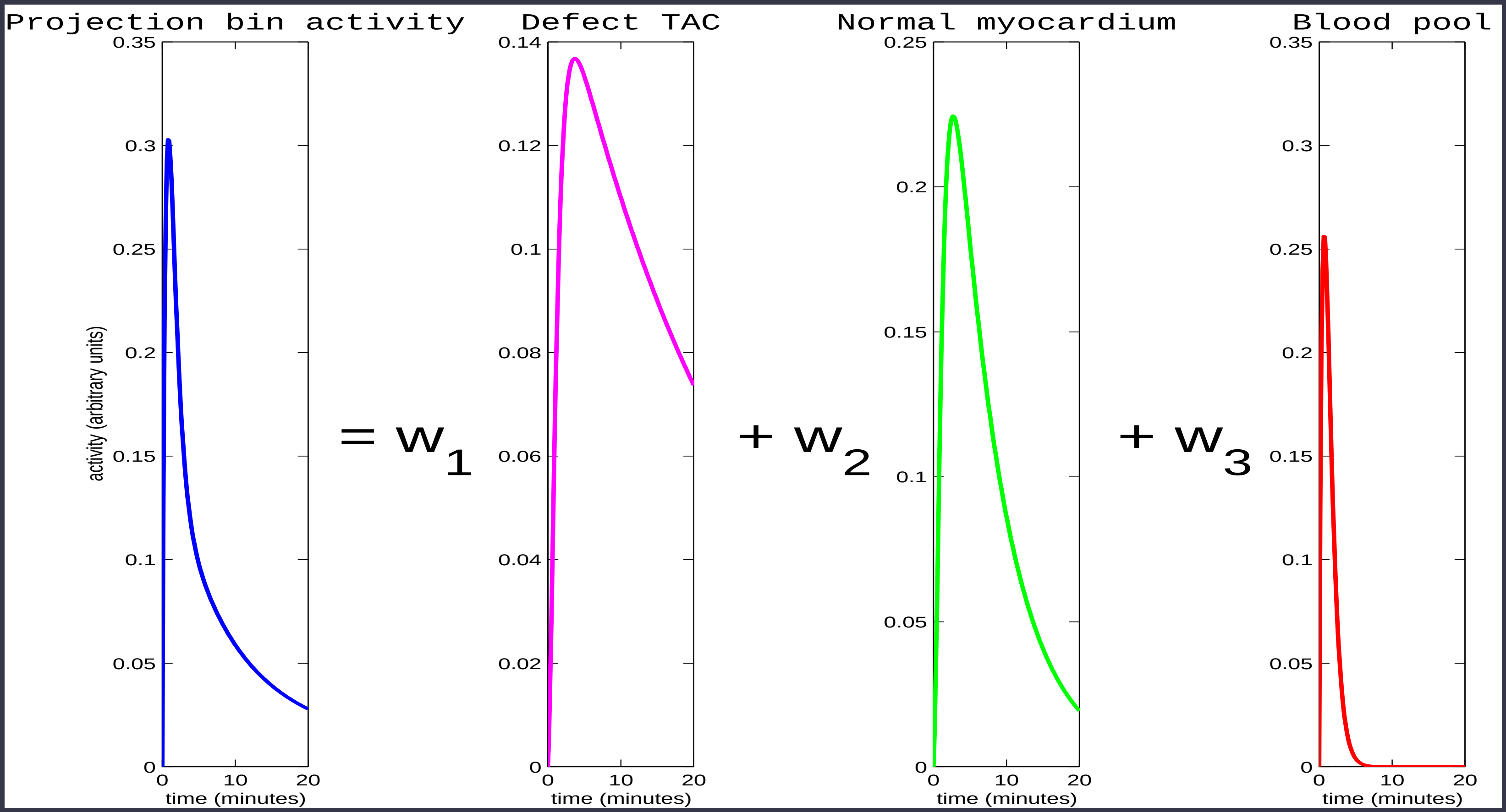
Extension to case of multiple regions is non-trivial:



Normal myocardium	Defect

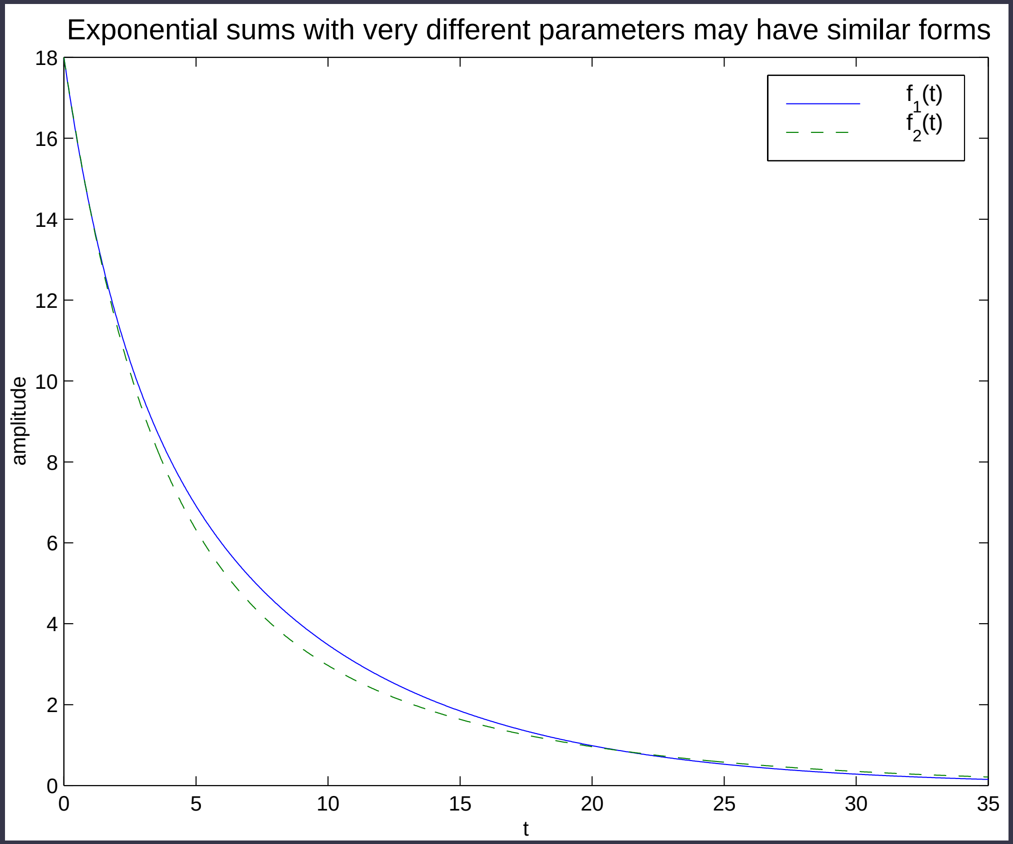






$$p(\theta(t), s) = i(t) * \underbrace{\sum_{m=1}^{3} w_m(\theta, s, k_1^m)\, e^{-k_2^m t}}_{\text{exponential sum}}$$

Parameter redundancy in exponential sums



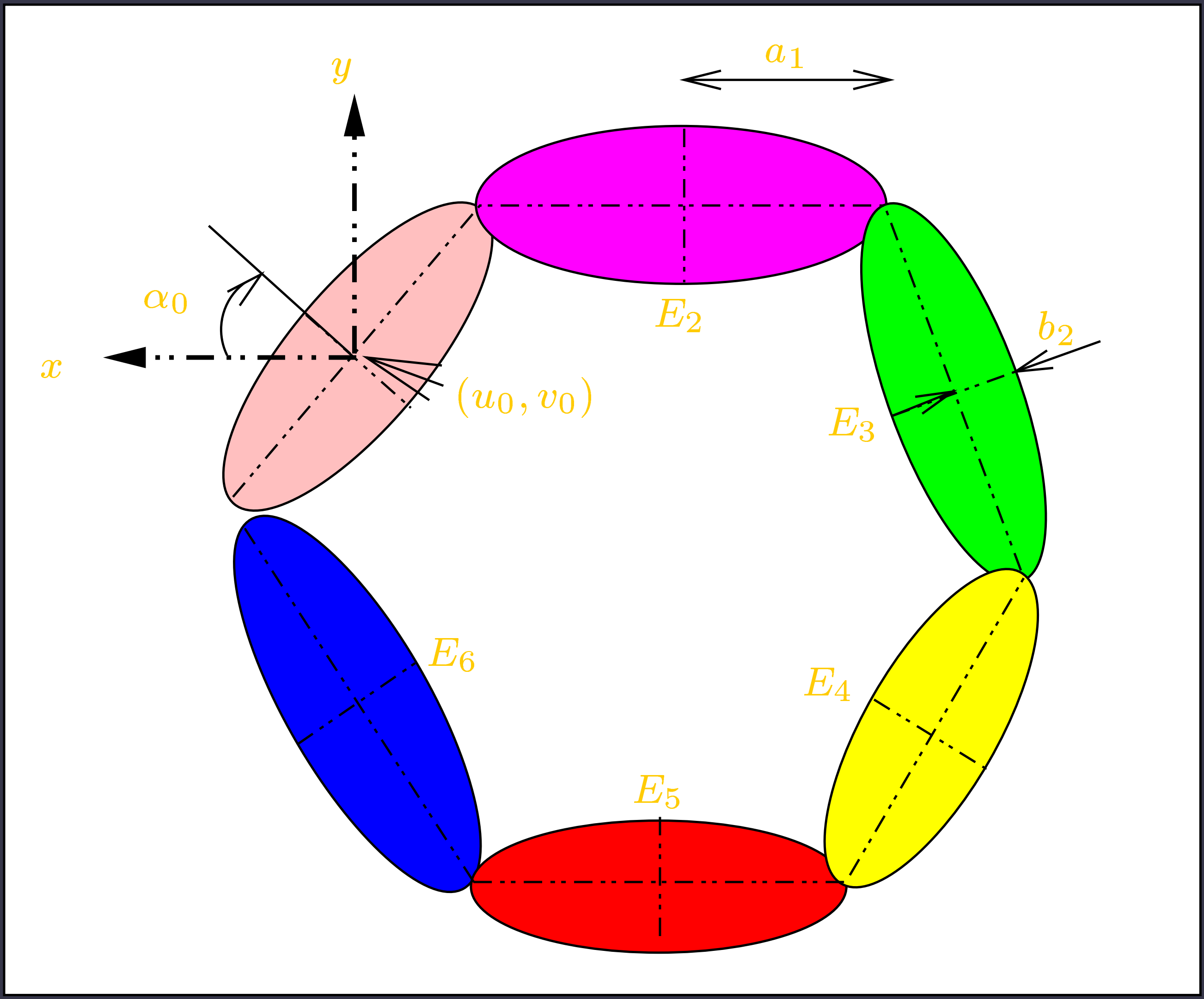
$$f_1 = 6\,\mathrm{e}^{-t/2} + 12\,\mathrm{e}^{-t/8}$$

$$f_2 = 11\,\mathrm{e}^{-t/3} + 7\,\mathrm{e}^{-t/10}$$

Chief objectives in present approach:

1. **Ensure robustness against noise:**
   - Employ highly constrained spatial model - cannot resolve conventional pixel-based image at low SNR's
   - Employ global-local hybrid optimization algorithm
2. **Improve condition of problem:**
   - Exponential sum problem: Construct new basis set for TAC's to reduce parameter redundancy
   - Constrain both geometric and kinetic parameters to feasible ranges
   - Reduce number of parameters

Non-overlapping ellipse ring model



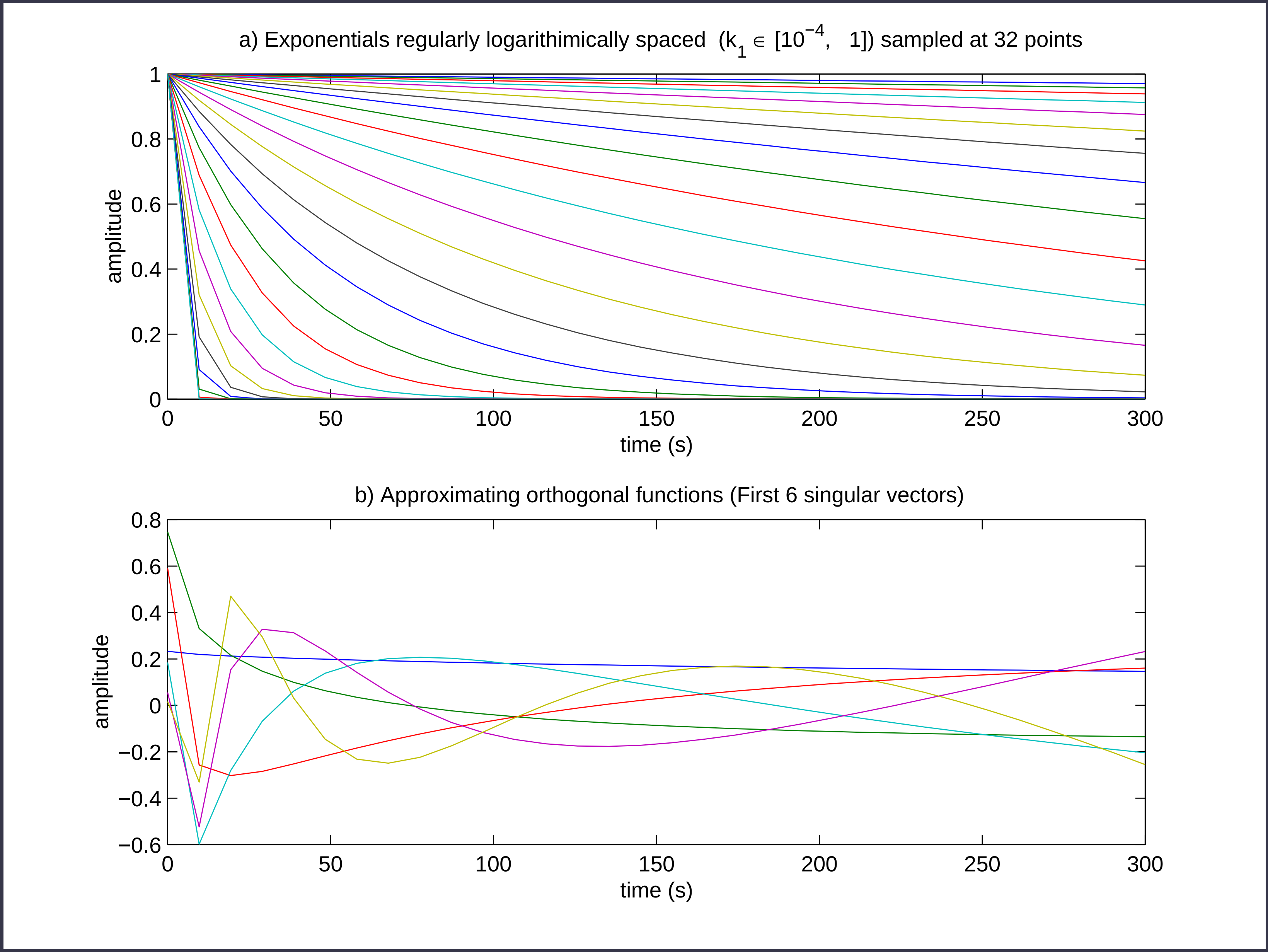
Orthogonal basis functions

1. Choose range of physiologically feasible rate constants:
   Here we choose $k_2 \in [0.001, 1]$ min^{-1}

2. Use singular value decomposition (SVD)
   to find an orthogonal basis for these exponentials

3. Choose first few principal components as approximating
   basis function set

4. Convolve these with the blood input function



a) Exponentials regularly logarithimically spaced ($k_1 \in [10^{-4}, 1]$) sampled at 32 points
amplitude
time (s)
0 0.2 0.4 0.6 0.8 1
0 50 100 150 200 250 300
b) Approximating orthogonal functions (First 6 singular vectors)
amplitude
time (s)
−0.6 −0.4 −0.2 0 0.2 0.4 0.6 0.8
0 50 100 150 200 250 300

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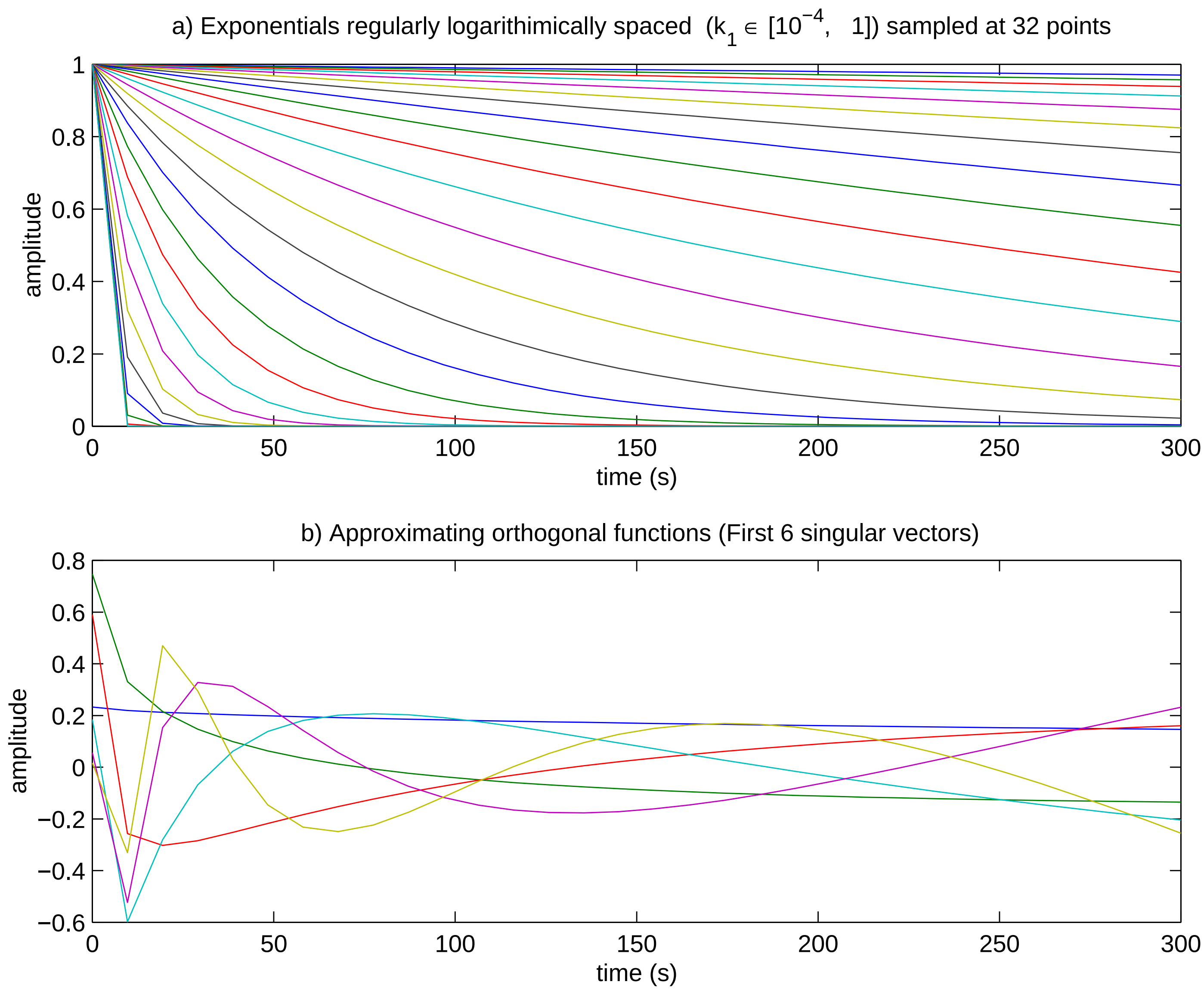
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amplitude
time (s)
b) Approximating orthogonal functions (First 6 singular vectors)
amplitude
time (s)

Original TAC composition for region n:

$$f_n(t) = \sum_{m=1}^{M} k_1^{mn}\, i(t) * \underbrace{\mathrm{e}^{-k_2^{mn}\, t}}_{\text{original exponential basis}}$$

Approximating discrete TAC:

$$\hat{f}_n[l] = \sum_{m=1}^{\tilde{M}} \mu^{mn}\, i(t) * \underbrace{u^m[l]}_{\text{orthogonal basis set}} \qquad l = 0,\, 1,\, \ldots,\, L-1.$$

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Under new basis we enjoy:

- Reduction in parameter redundancy

  $\Rightarrow$ Better conditioned problem

- Dimensionality reduction ($\tilde{M} \ll M$)

  $\Rightarrow$ Skirts 'curse of dimensionality'

  $\Rightarrow$ Lowers computational cost

BUT

TAC's may now be negative - need to constrain the μ^{mn}

Estimation problem:

Simultaneously determine:

- Parameters of ellipse ring model
- Parameters of the compartmental model of each region

Given:

- The measured sinogram
- A rough initial placement of the ring within image space

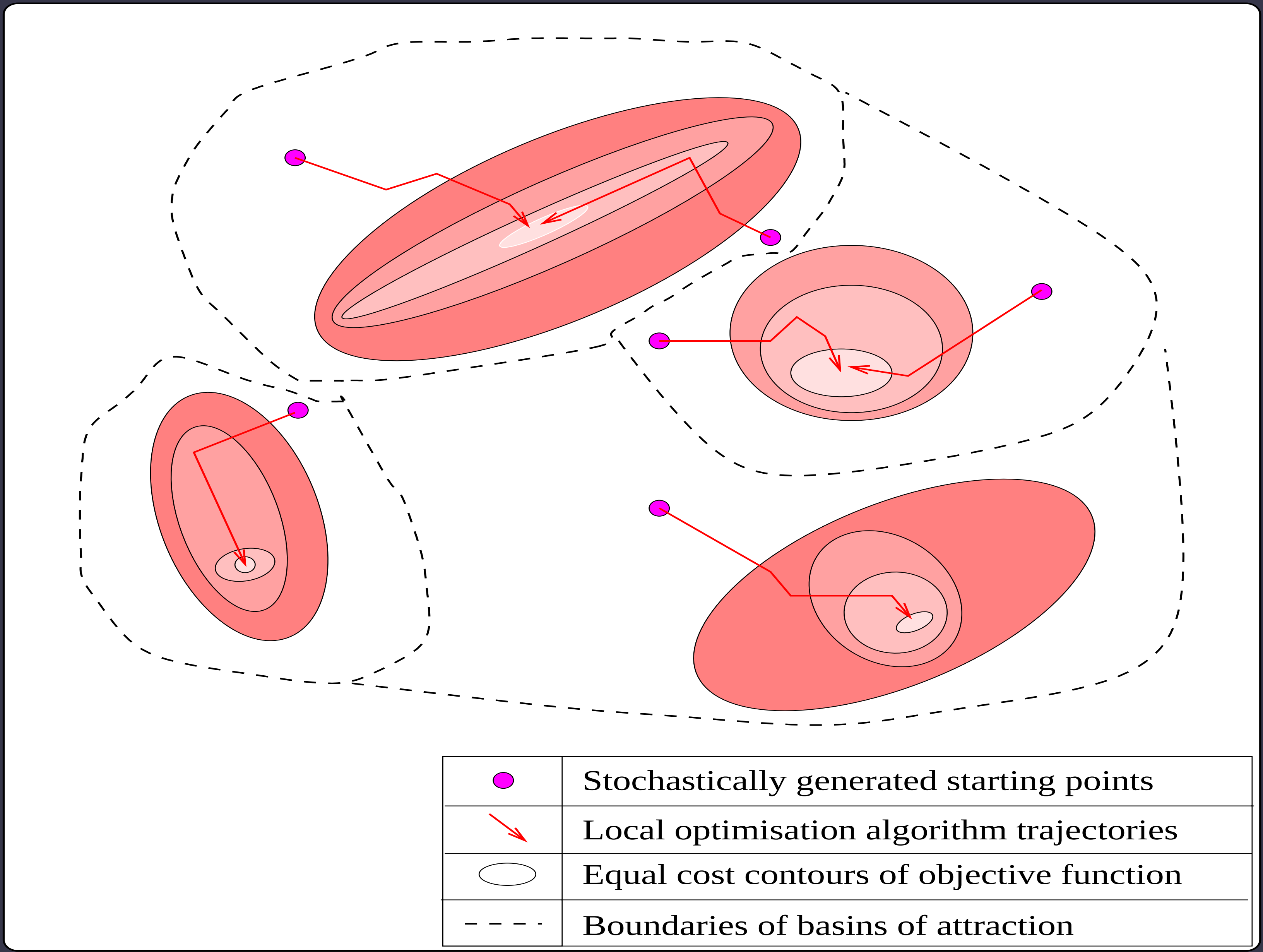
Hybrid optimization algorithm

Least squares cost function has thousands of suboptimal local minima

$\Rightarrow$ Multistart algorithm is **essential**

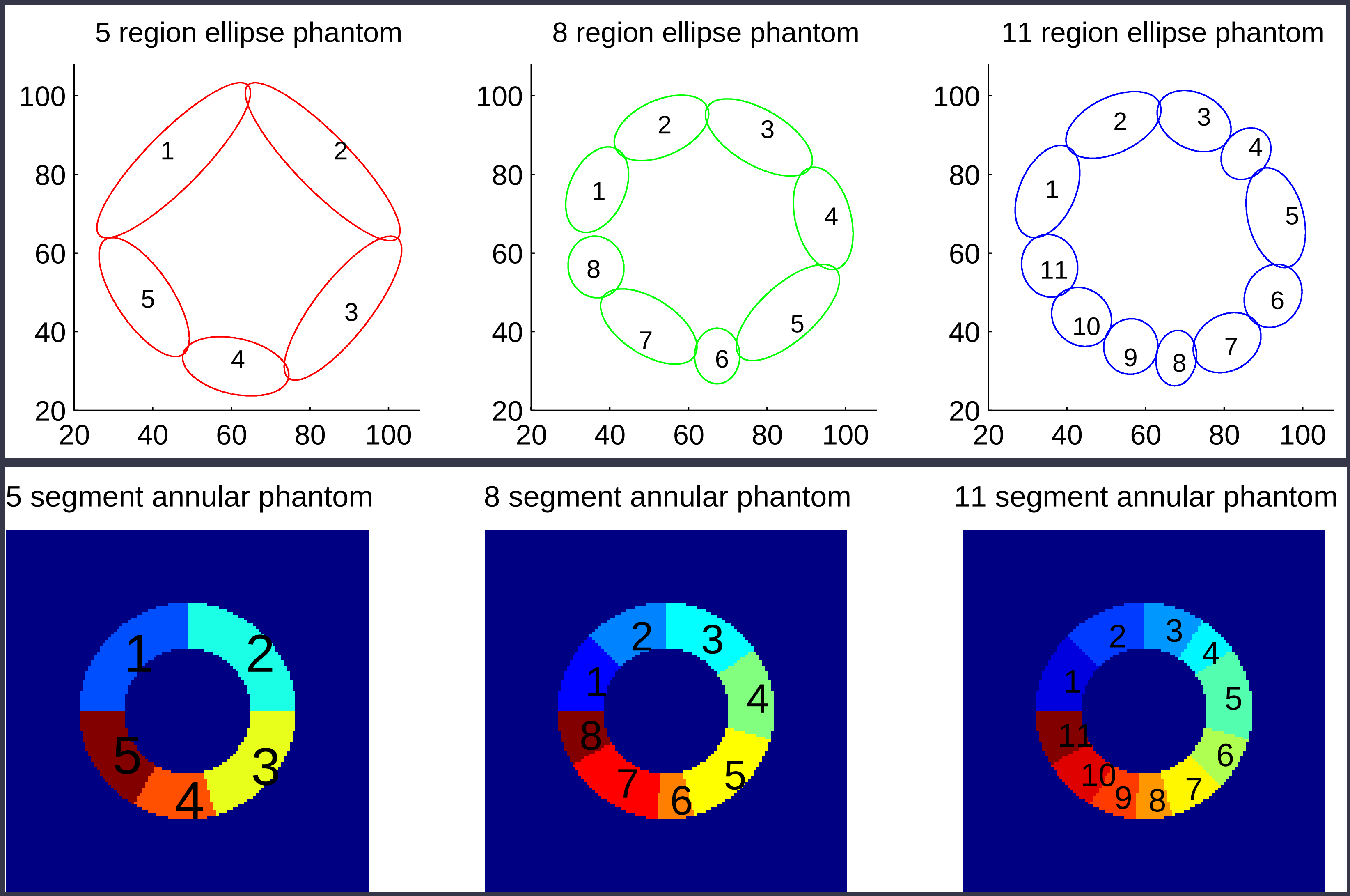
Starting point generation: Adaptive simulated annealing (ASA) algorithm

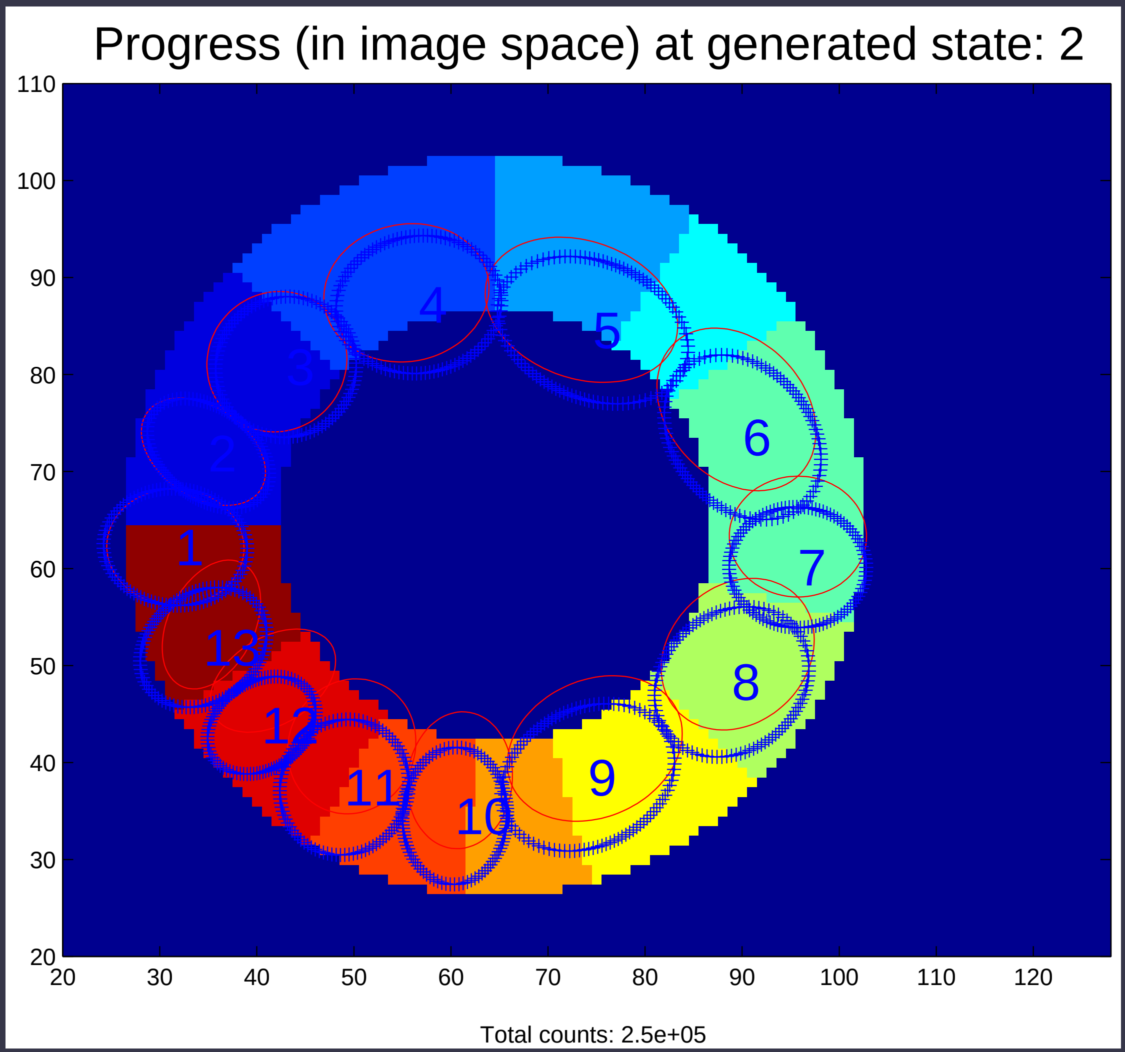
Parameter value refinement: Projected descent algorithm - interval bounds on parameters

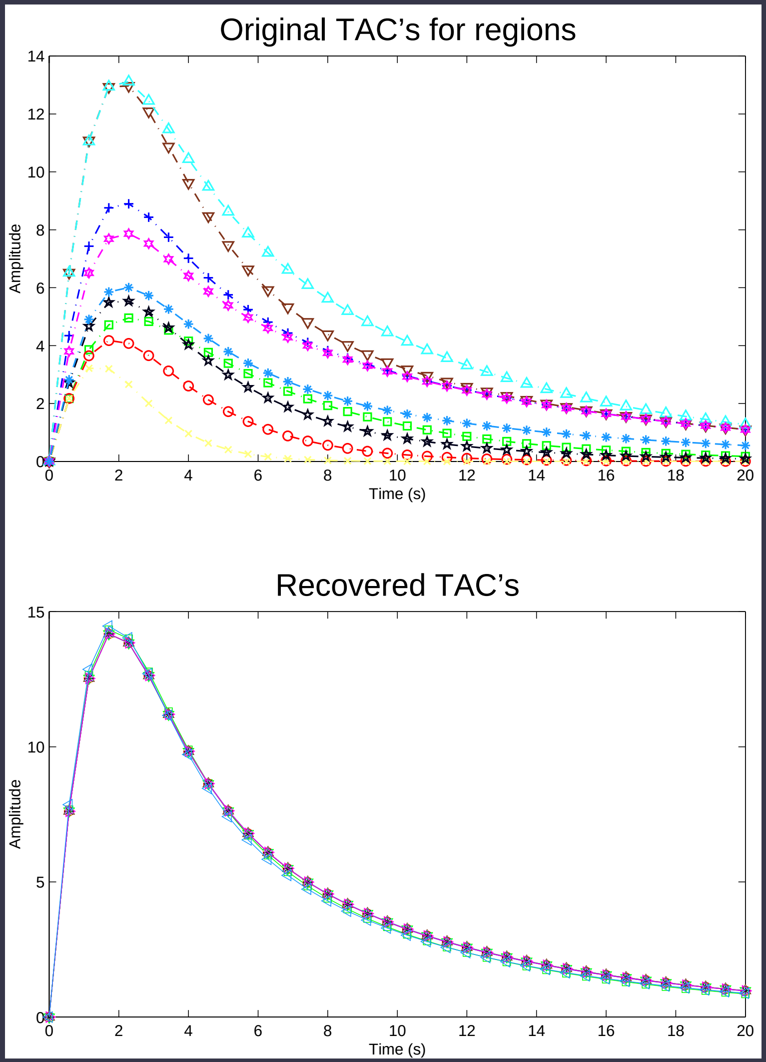

Stochastically generated starting points
Local optimisation algorithm trajectories
Equal cost contours of objective function
Boundaries of basins of attraction

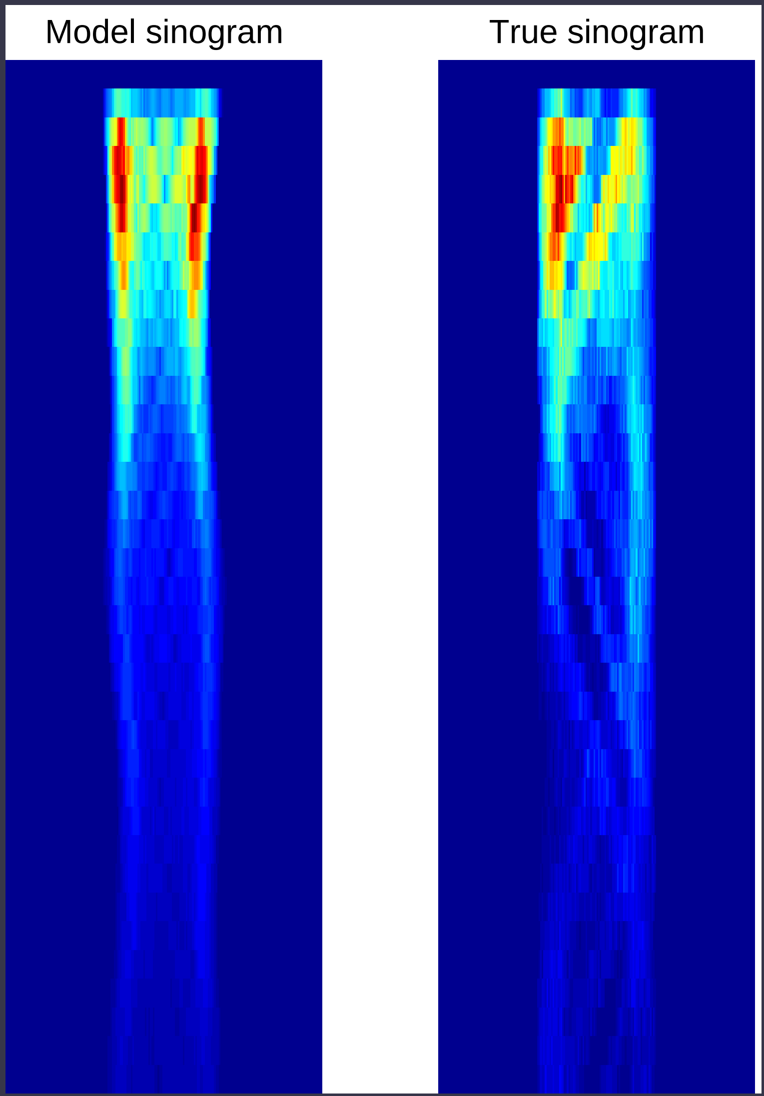
Algorithm evaluation

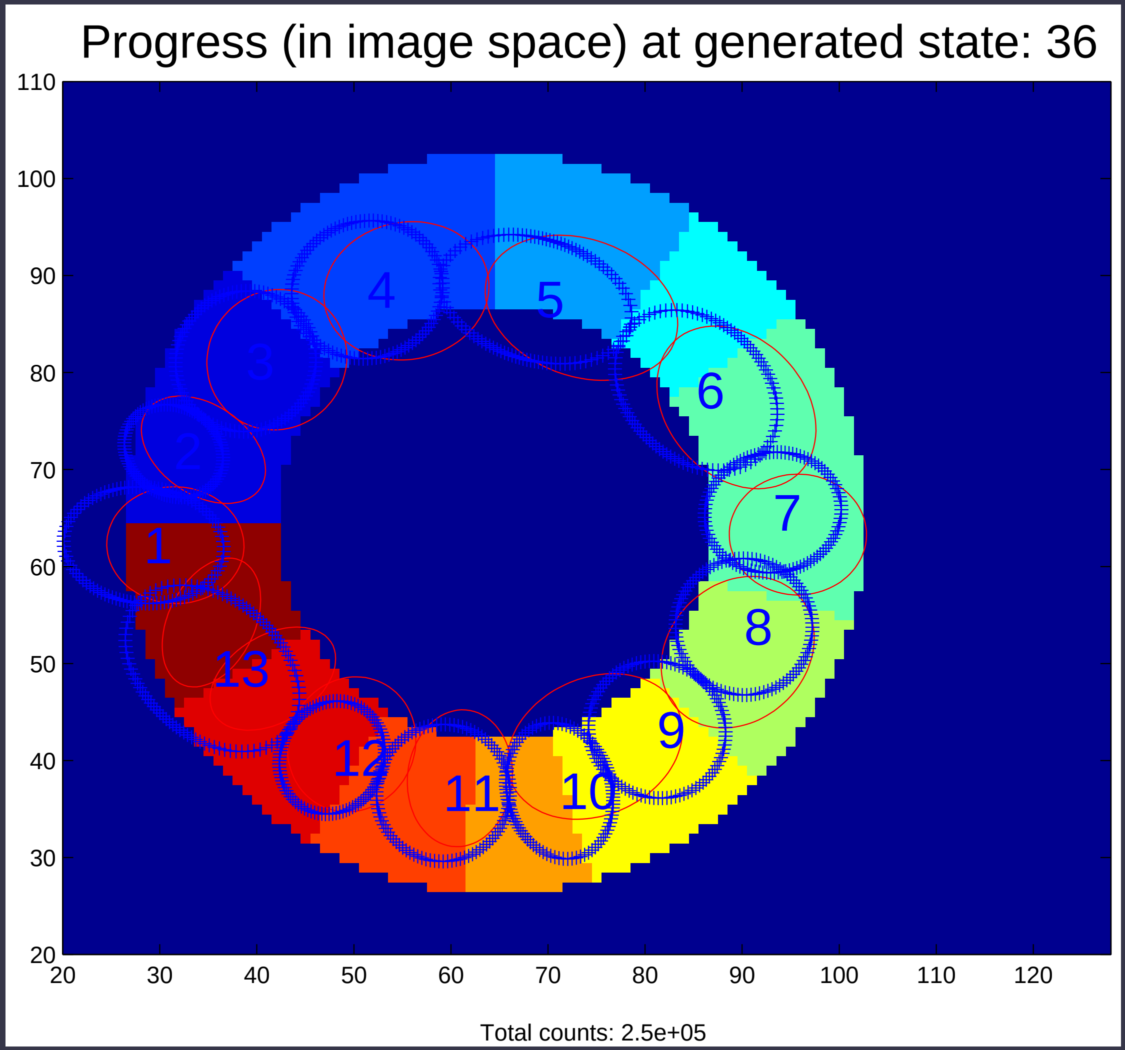
- Generated $250,000$ photon counts per slice
- Ignore attenuation, scatter & imperfect system response
- Fit to sinogram of segmented annulus phantom -
  Can ellipse model cope with spatial model mismatch?
- Include exaggerated number of compartments in phantom


5 region ellipse phantom
8 region ellipse phantom
11 region ellipse phantom
5 segment annular phantom
8 segment annular phantom
11 segment annular phantom


Progress (in image space) at generated state: 2
110
100
90
80
70
60
50
40
30
20
20
30
40
50
60
70
80
90
100
110
120
1
2
3
4
5
6
7
8
9
10
11
12
13
Total counts: 2.5e+05


Original TAC's for regions
Amplitude
Time (s)
Recovered TAC's
Amplitude
Time (s)


Model sinogram
True sinogram



Progress (in image space) at generated state: 36
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40
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60
70
80
90
100
110
120
1
2
3
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7
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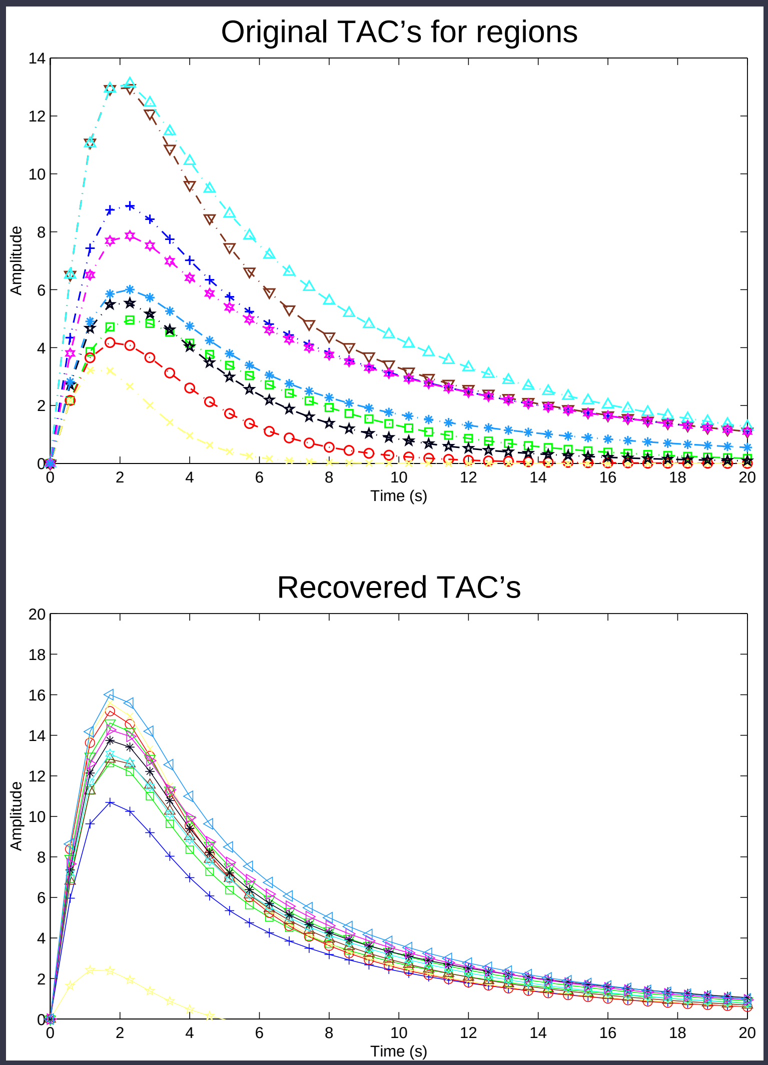
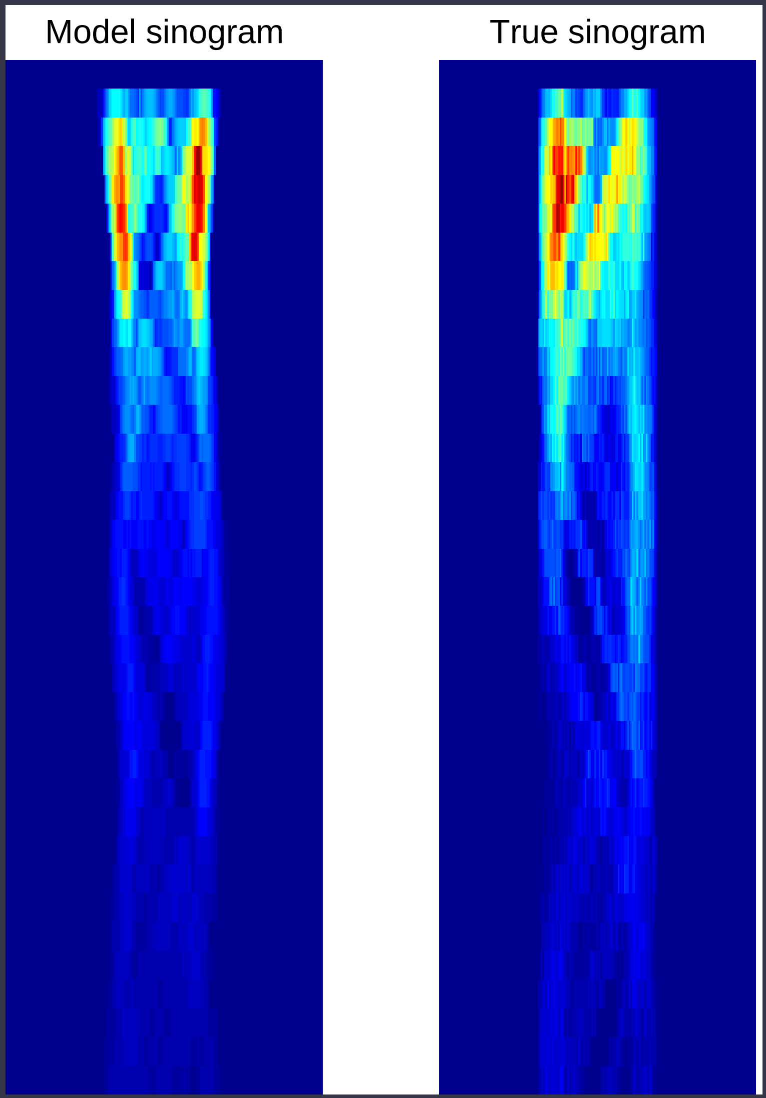
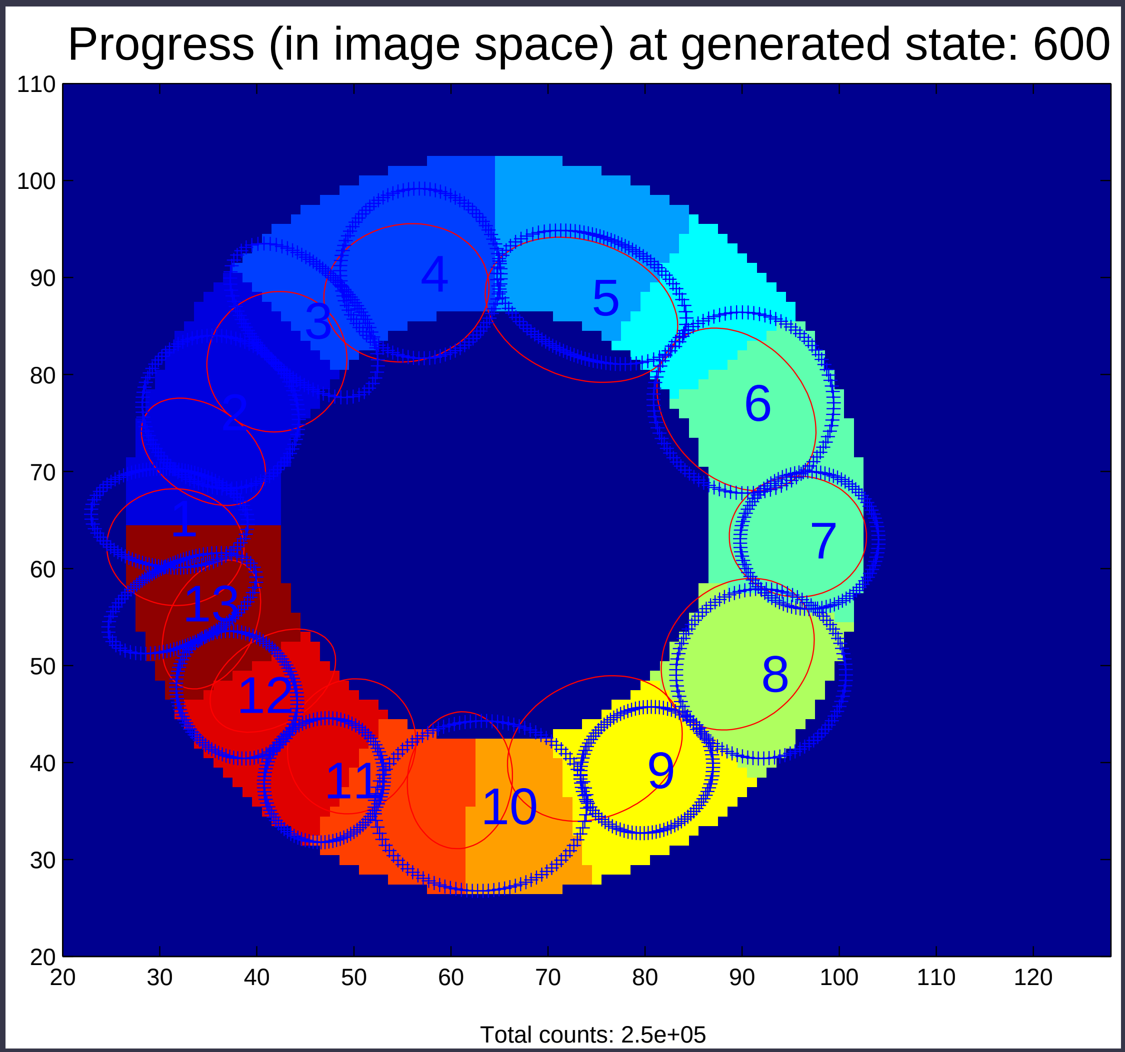
9
10
11
12
13
Total counts: 2.5e+05


Original TAC's for regions
Amplitude
Time (s)
Recovered TAC's
Amplitude
Time (s)


Model sinogram
True sinogram


Progress (in image space) at generated state: 600
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60
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60
70
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90
100
110
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11
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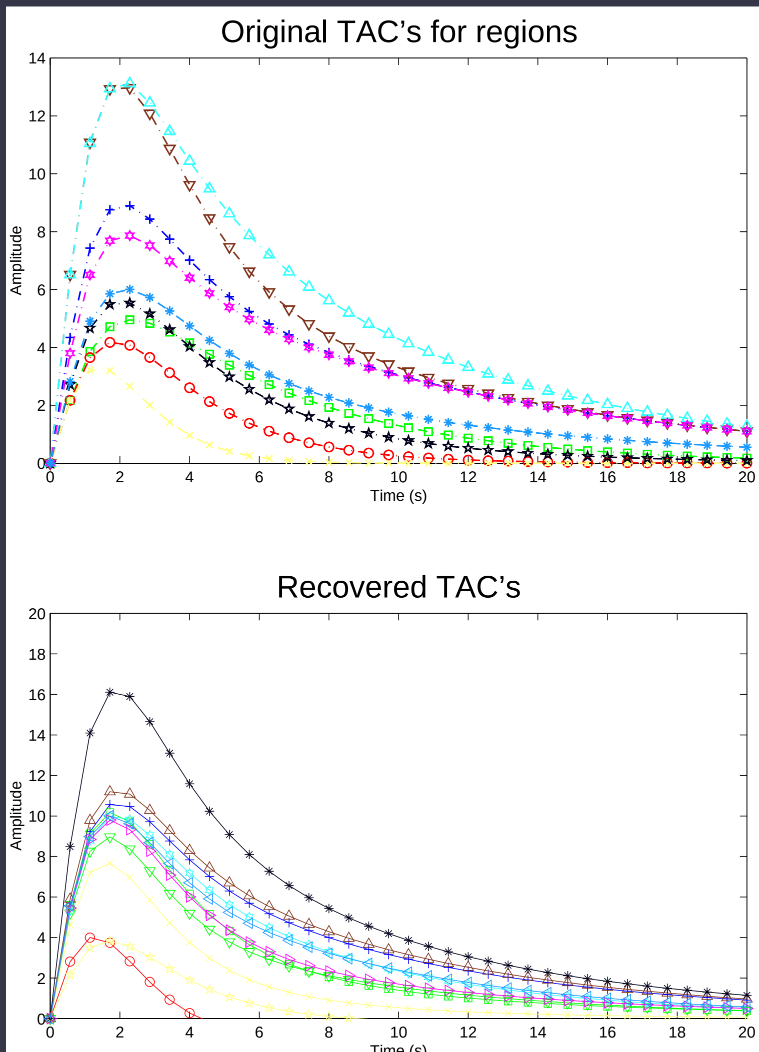
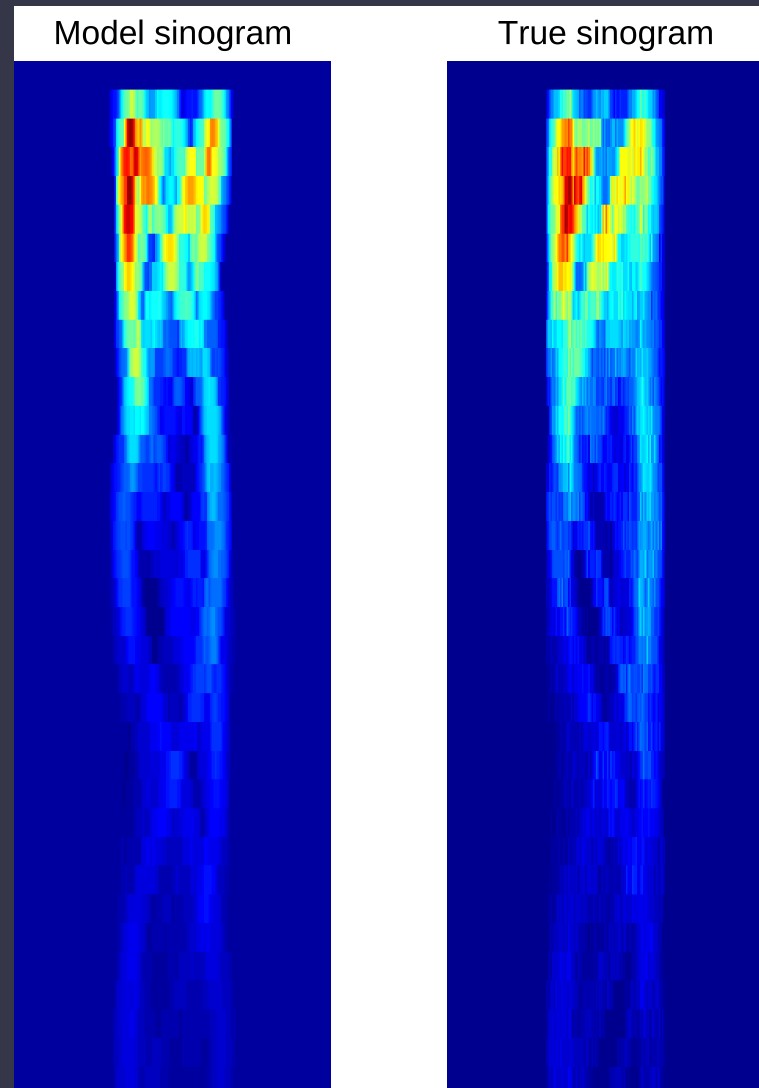
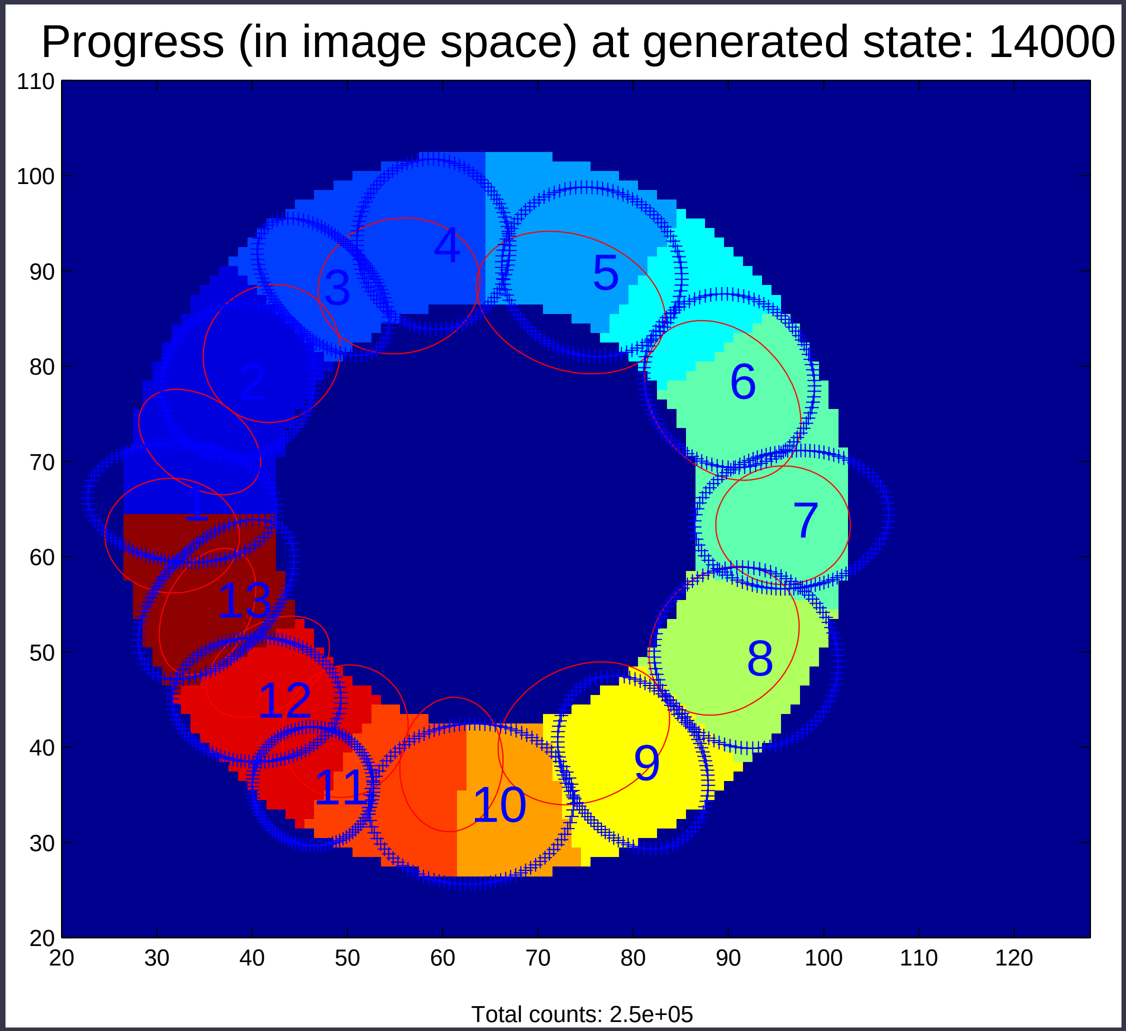
13
Total counts: 2.5e+05


Original TAC's for regions
Amplitude
Time (s)
Recovered TAC's
Amplitude
Time (s)


Model sinogram
True sinogram



Progress (in image space) at generated state: 14000
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100
90
80
70
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20
20
30
40
50
60
70
80
90
100
110
120
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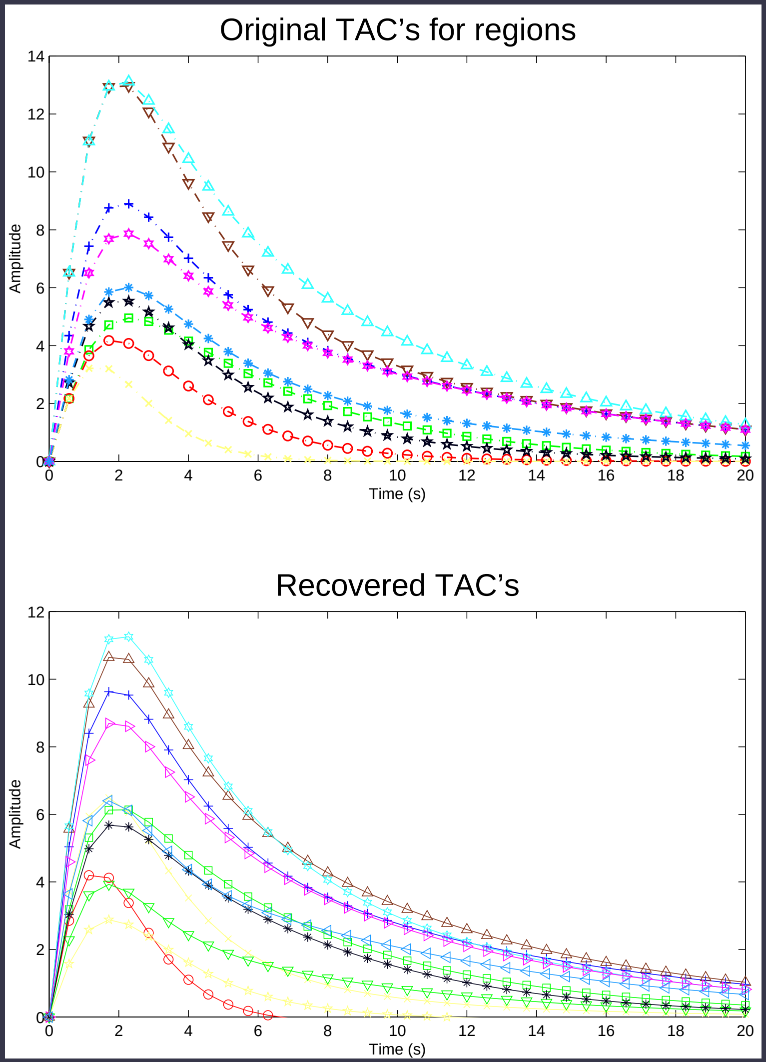
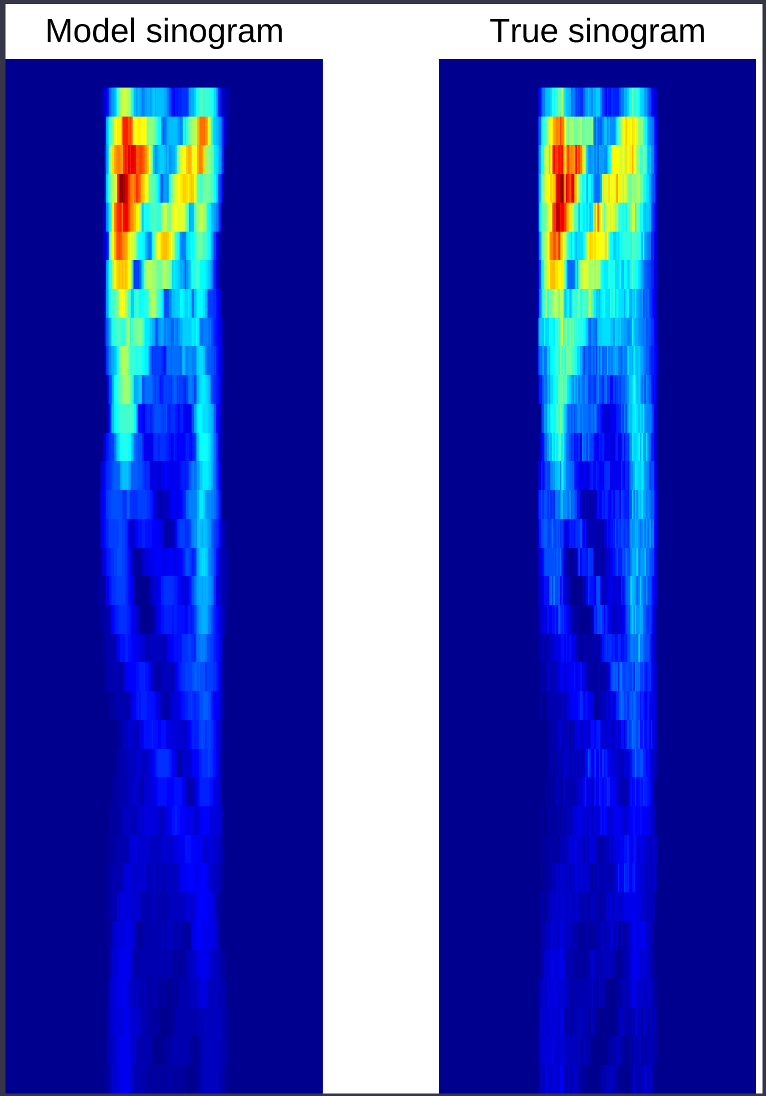
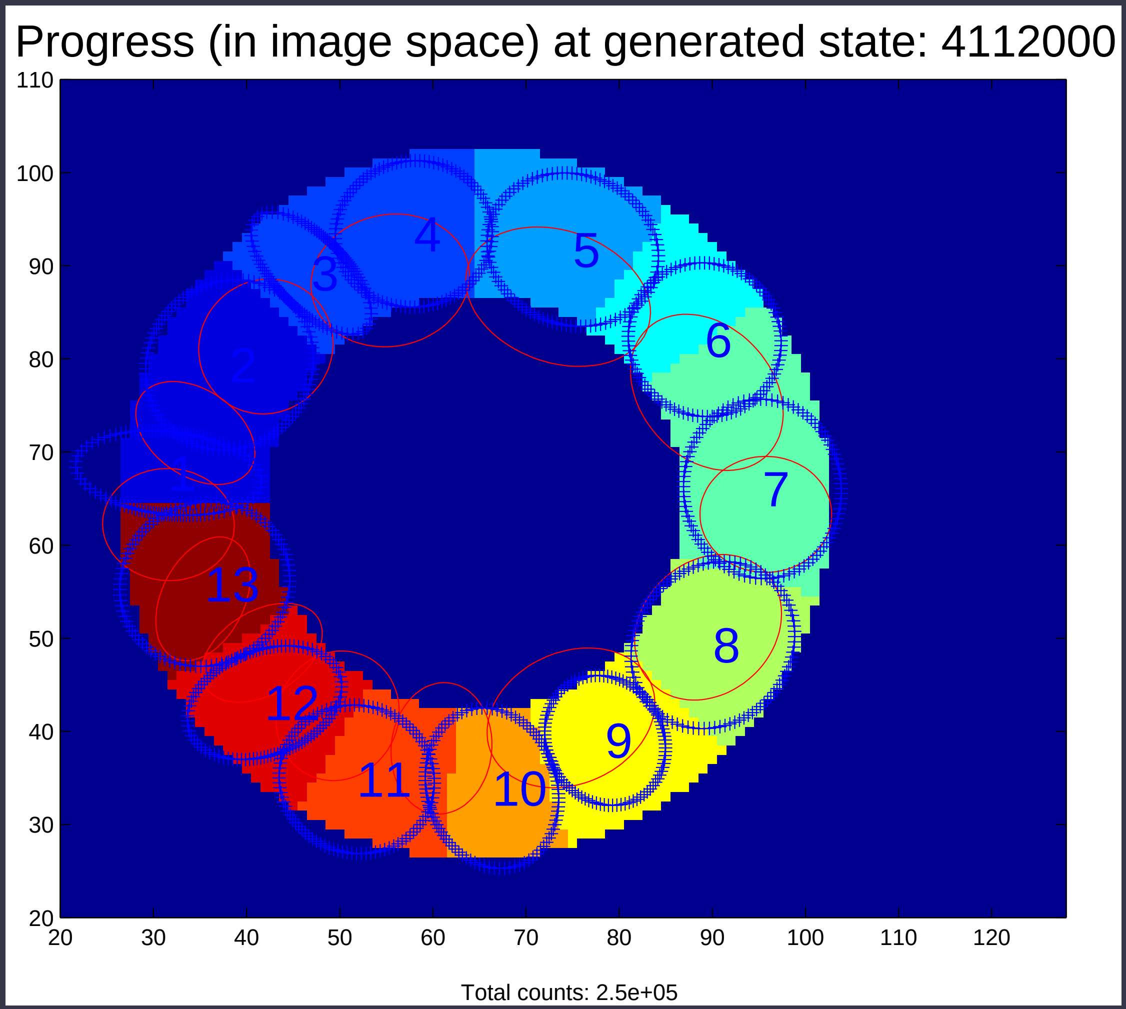
Total counts: 2.5e+05


Original TAC's for regions
Amplitude
Time (s)
Recovered TAC's
Amplitude
Time (s)


Model sinogram
True sinogram


Progress (in image space) at generated state: 4112000
Total counts: 2.5e+05
1
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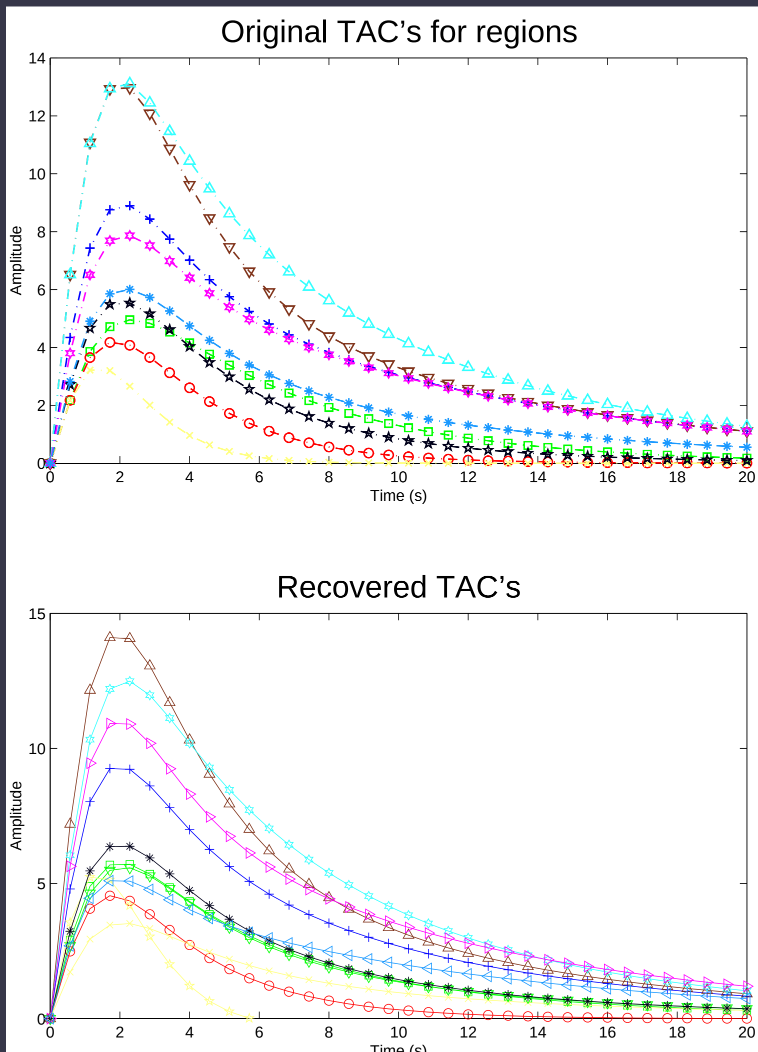
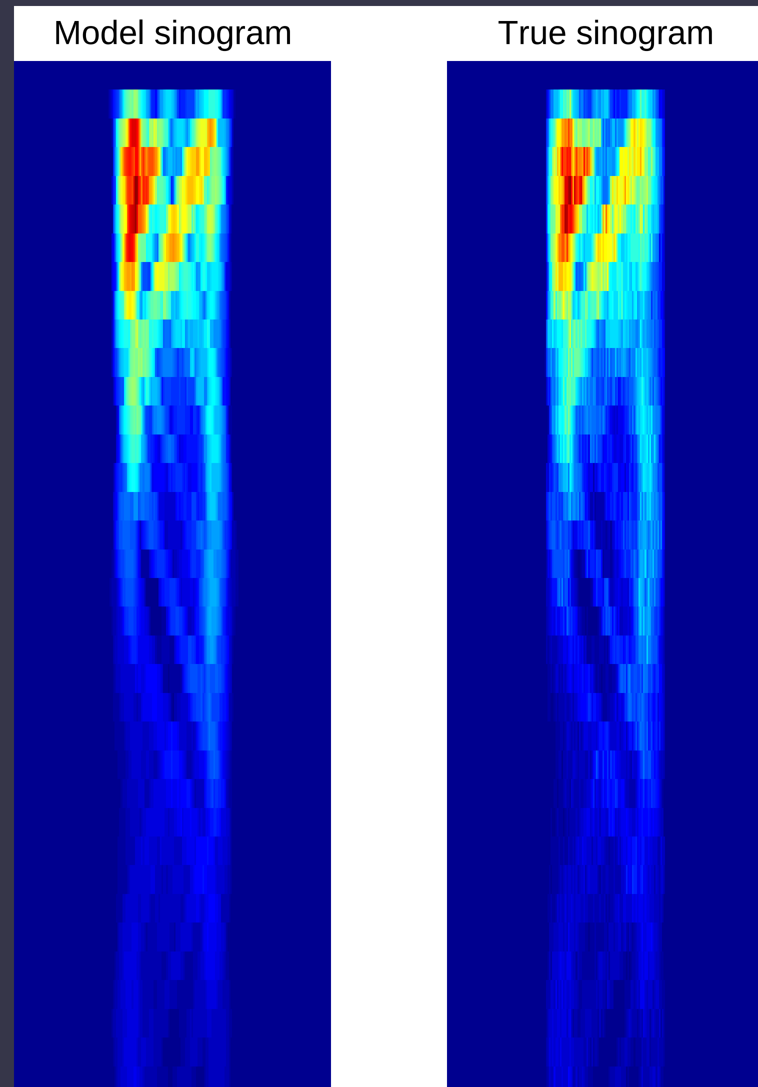

Original TAC's for regions
Amplitude
Time (s)
0
2
4
6
8
10
12
14
16
18
20
Recovered TAC's
Amplitude
Time (s)
0
5
10
15


Model sinogram
True sinogram

Results

- This particular configuration tested over 20 noise realizations
- Good visual fits are obtained
- Adjacent regions with similar TAC's may be merged

Metric	Value $\pm$ std. dev.
Mean RMS deviation in recovered TAC's:	6.6 $\pm$ .9%
Mean RMS error in wash-in parameter (k_1):	20.8 $\pm$ 4.7%
Mean RMS error in wash-out parameter (k_2):	29.6 $\pm$ 4.7%

Conclusions

- Performance is encouraging - estimates of useful accuracy are obtained
- Execution is slow (typically 12 hours on Pentium II 400MHz) $\Rightarrow$ need to parallelize algorithm
- Noise and model mismatch problems are minor compared to problem of extricating individual TAC's from their sums
- Algorithm is believed to be the first to successfully perform simultaneous spatial and kinetic estimation on multiple regions at realistic noise levels
- How will non-ideal imaging conditions affect performance?